COTMANHAY
JUNIOR SCHOOL

Calculations Policy
2023

This calculation policy has been written to equip all learners, staff and parents with the guidance they will need in order to assist children with the appropriate mental and standard written calculation strategies they are using in class. This policy ensures consistency and progression in line with the National Curriculum for Mathematics.

At Cotmanhay Junior School, we use the 'White Rose Hub' format as a basis for our planning. White Rose follows the **Concrete – Pictorial – Abstract** approach of teaching maths.

- **Concrete** is the 'doing' stage; using concrete resources to solve problems.

- The **Pictorial** or 'seeing' stage uses representations of objects to solve problems. This helps children to make the connection between the physical object and abstract levels of understanding.

- The **Abstract** stage brings in mathematical symbols, such as +, -, x, ÷ to indicate addition, subtraction, multiplication and division.

The emphasis, initially, is on mental calculation skills but progresses to written strategies that children are expected to become fluent with. The standards within this document have been arranged into age-related expectations for each year group and we expect the majority of children to achieve them. However, it is important to recognise that children develop their mathematical skills at different rates and learning should be pitched at a level that is suitable to their needs.

It is our aim that all our children have a secure understanding of standard written methods for all four number operations by the end of LKS2 so that the methods can be consolidated and extended throughout Year 5. In Year 6, children are able to focus on application of skills and progression onto more challenging learning.

Contents Page:

Addition

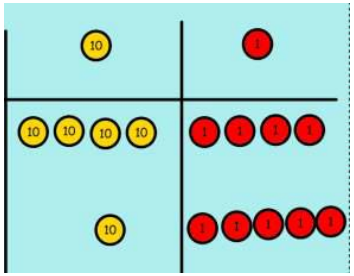
- Children should understand that addition is commutative and, therefore, calculations can be rearranged. For example, **4 + 13 = 17 is the same as 13 + 4 = 17.**
- Ensure that children understand the = sign means ***'is the same as'*** as opposed to '*makes*'.
- Ensure children see calculations where the equals sign is in a different position. For example, **3 + 2 = 5** and **5 = 3 + 2**

Year 3	Year 4	Year 5	Year 6
Age-related expectations: • Add and subtract numbers mentally, including: ***A three-digit number and ones*** ***A three-digit number and tens*** ***A three-digit number and hundreds*** • Add and subtract numbers with up to 3 digits, using formal written methods (columnar). • Estimate answers to calculations; use inverse to check. • Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction	**Age-related expectations:** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate. • Estimate and use inverse operations to check answers to a calculation. • Solve two-step addition and subtraction problems in context, deciding which operations and methods to use and why.	**Age-related expectations:** Add and subtract whole numbers with more than 4 digits using formal written methods of columnar addition and subtraction. • Add and subtract mentally with increasingly larger numbers. • Use rounding to check answers and determine, in the context of a problem, levels of accuracy. • Solve multi-step addition and subtraction problems in context, deciding which operations and methods to use and why.	**Age-related expectations:** • Use knowledge of the order of operations to carry out calculations involving all four operations. • Solve multi-step addition and subtraction problems in context, deciding which operations and methods to use and why. • Solve complex problems involving addition and subtraction. • Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Vocabulary: +, add, addition, more, plus, make, sum, total, altogether, score, double, near double, One more, two more, ten more, one hundred more, How many more make…? How many more is… than…? How much more is…? Is the same as =, equals, Tens column, hundreds column Number bonds, pairs, facts	**Vocabulary:** Add, addition, more, plus, increase, sum, total, altogether, score, double, near double, How many more to make…? =, equals, …is the same as… Tens column, hundreds column, Inverse One more, two more, ten more, one hundred more, Number bonds, facts, pairs	**Vocabulary:** Add, addition, more, plus, increase, sum, total, altogether, score, double, near double, How many more to make…? =, equals, …is the same as… Ones column, tens column, hundreds column, tenths column, Inverse One more, two more, ten more, one hundred more, Number bonds, facts, pairs	**Vocabulary:** Add, addition, more, plus, increase, sum, total, altogether, score, double, near double, How many more to make…? =, equals, …is the same as… Ones column, tens column, hundreds column, tenths column, hundredths column, Inverse One more, two more, ten more, one hundred more, Number bonds, facts, pairs

Children will use tactile equipment to support them in their calculations. They can record their calculations using their own diagrams and drawings. For example, lines for 10 sticks and dots for ones blocks.

e.g. 34 + 23 = 57

Or, they can make numbers on a place value grid.



Children would need to add the least significant digits first in preparation for the formal written method.

When the ones total more than 10, children should be encouraged to **exchange** for a 10 and ones. This is the start of children understanding 'carrying' in formal addition.

Based on their experiences in Year 3, children will continue to add using a formal written method, adding up to 4-digit numbers, exchanging from all digits.

Th	H	T	O
	1	3	6 8
+	3	6	5 3
	5	0	2 1
	1	1	1

	£3.48
+	£0.78
	£4.26
	1 1

Using similar methods, children will:

- Add several numbers with a varying amount of digits.
- Begin to add two or more 3-digit sums of money- with or without the adjustment from the pence to the pounds
- Know that decimal points should line up under each other (in their own square); particularly when adding mixed amounts. For example: **£3.59 + 78p**

Children should extend the exchanging method to numbers with four or more digits.

	3	5	8 7
+	6	6	7 5
1	0	2	6 2
1	1	1	

Using formal written methods, children will:

- Add several numbers with varying amounts of digits.

3	1	2	1
		3	7
+		1	4 8
3	3	0	6

- Begin to add two or more decimal fractions with up to 3-digits and the same number of decimal places.
- Know that decimal points should line up under each other (in their own square); particularly when adding mixed amounts. For example: **3.2m + 280cm**

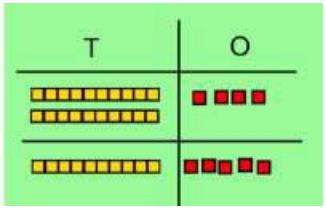
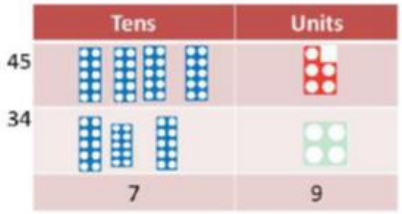
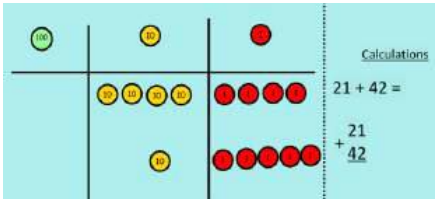
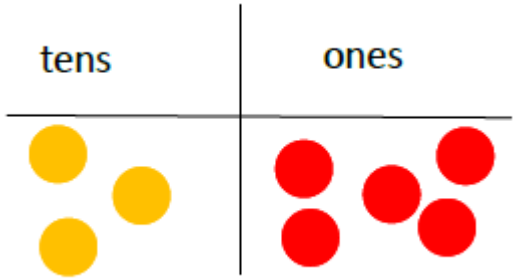
	3.20
+	2.80
	6.00
	1

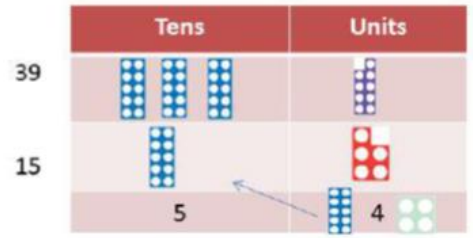
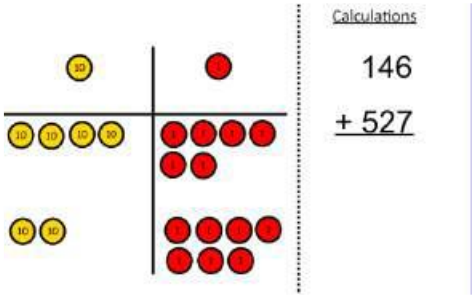
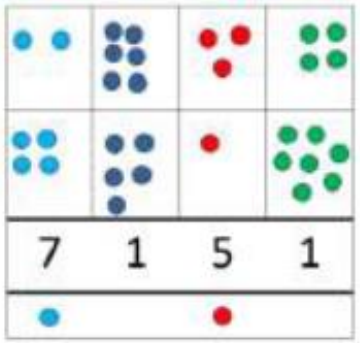
Children should extend the exchanging method to numbers with any number of digits.

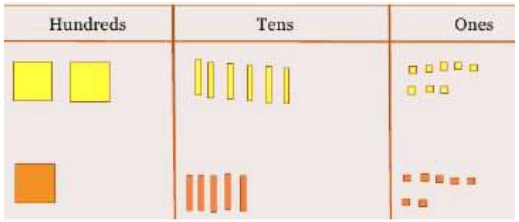
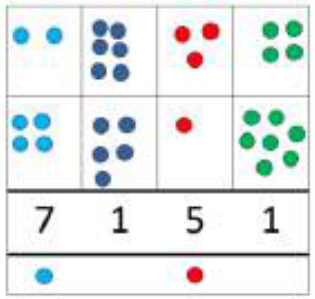
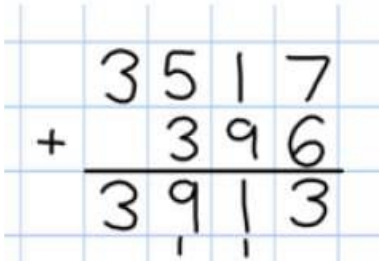
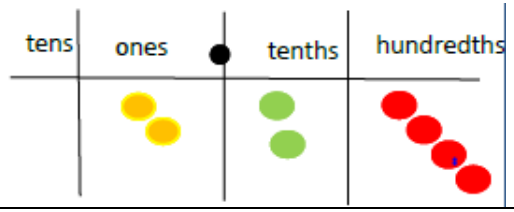
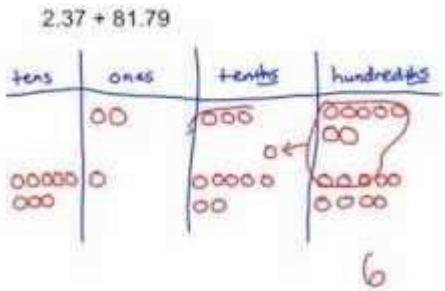
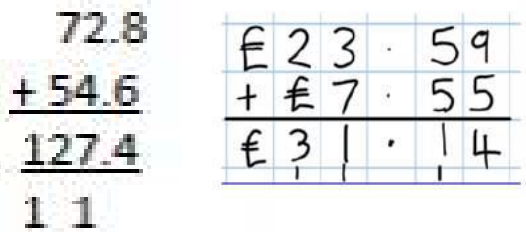
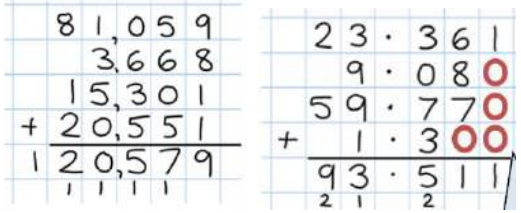
Using similar methods, children will:

- Add several numbers with various numbers of digits.
- Begin to add two or more decimal fractions with up to four digits and up to three decimal places.
- Know that decimal points should line up under each other (in their own square); particularly when adding mixed amounts. For example: **401.2 + 26.859 + 0.71**

Continue using Base 10, and other practical apparatus, to support with the transition into a formal written method of addition. Children should add the least significant digits first in preparation for the formal written method. TO 67 + 24 91 Leading to: H T U 2 6 7 + 8 5 3 5 2 Practical resources can be used alongside to model the transition to the formal written method but there is no expectation for this to be recorded by the children.			

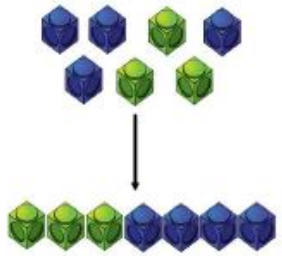
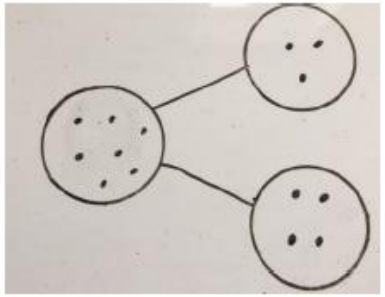
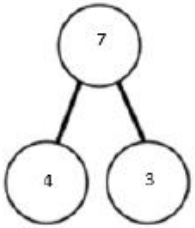
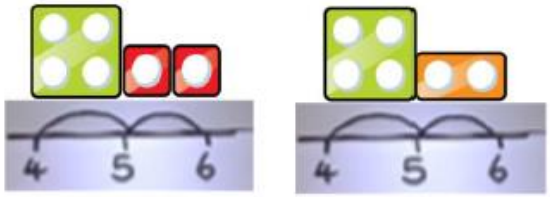
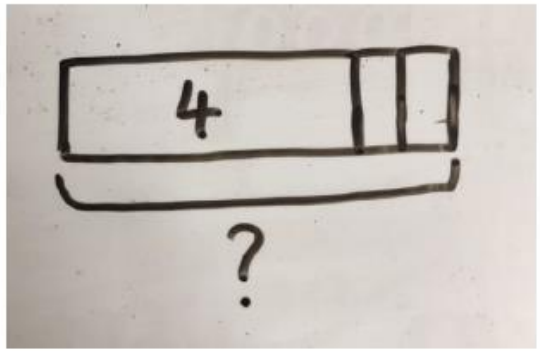
Objective and strategy	Concrete	Pictorial	Abstract
Y3 Column addition- no exchanging (friendly numbers) Add two or three 2 or 3-digit numbers	Model using practical apparatus (Dienes or Numicon)  Add the ones first, then the tens  Move on to using place value counters  	Children move to drawing the counters using a tens and ones frame.  	Add the ones first, then the tens, then the hundreds. 2 2 3 + 1 1 4 3 3 7

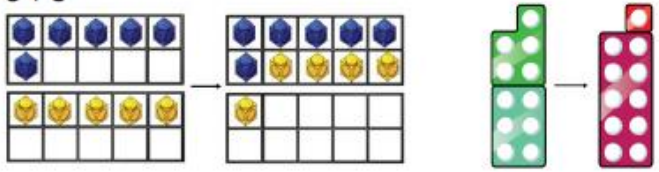
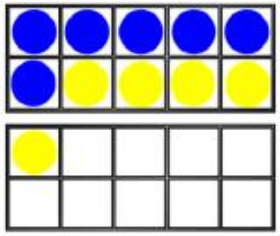
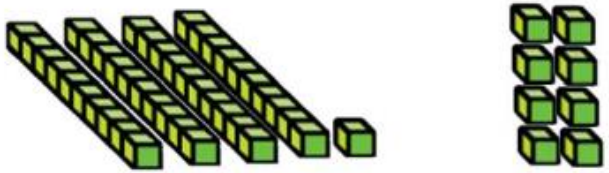
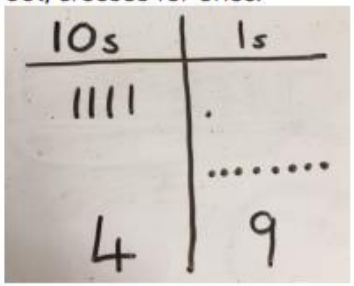
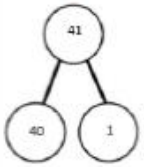
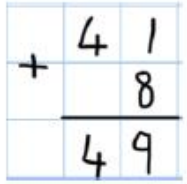
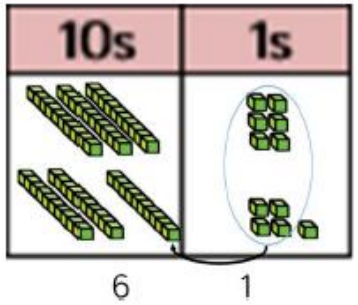
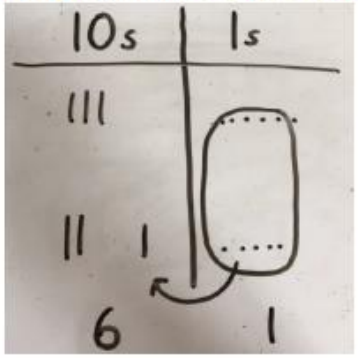
Column addition with exchanging.	Exchange ten ones for a ten. Model using practical apparatus (Dienes, PV counters, Numicon)   	Children can draw a representation of the grid to further support their understanding; exchanging the ten **underneath** the line.  	Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1

Objective and strategy	Concrete	Pictorial	Abstract
Y4 Add numbers with up to 4-digits	Children continue to use practical resources (Dienes or PV counters) to add, exchanging tens ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using a Place Value grid. 	Continue form previous work to exchange hundreds as well as tens. Relate to money and measures. 
Y5 Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money	As Year 4 Introduce decimal place value counters and model exchange for addition. 	As Year 4 	As Year 4 
Y6 Add several numbers of increasing complexity; including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders. 

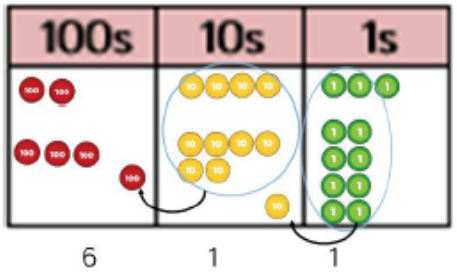
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

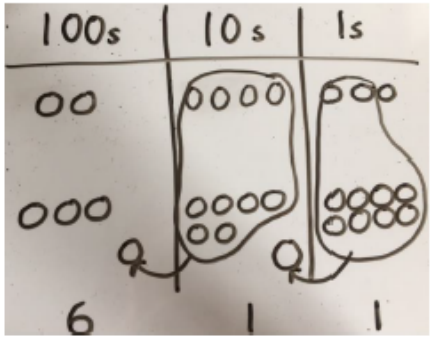
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10. 36 + 25= 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: +25 61 1

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

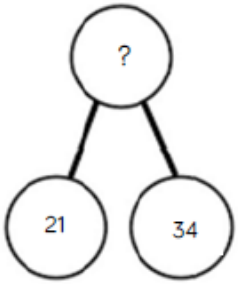


Chidren to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

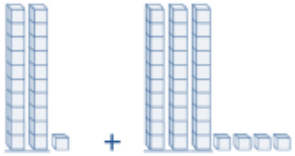
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

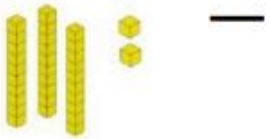
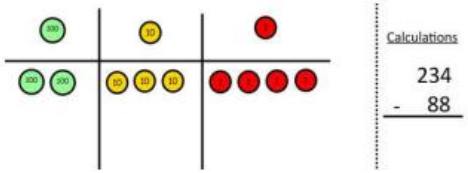
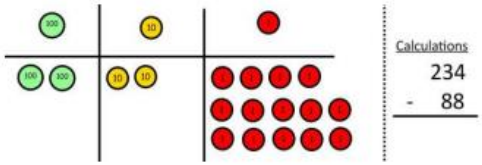
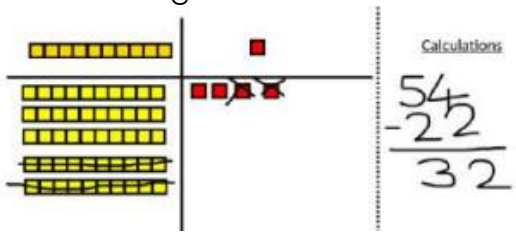
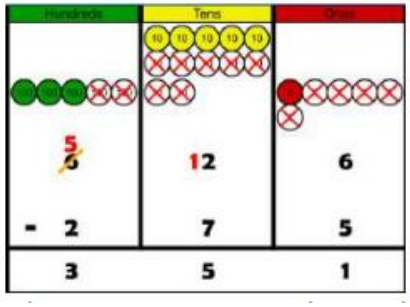
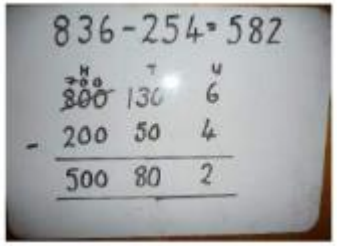
10s	1s
10 10	1
10 10 10	?
?	5

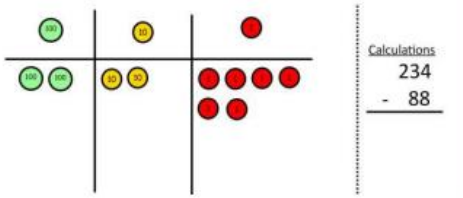
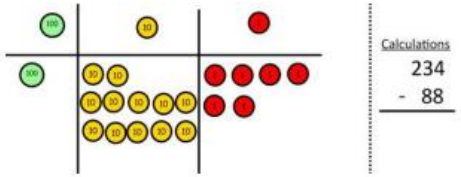
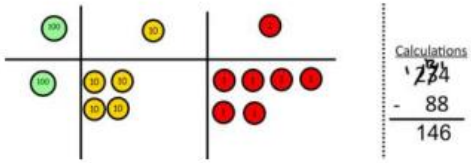
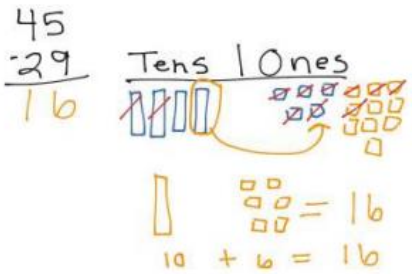
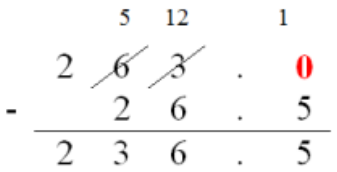
Subtraction

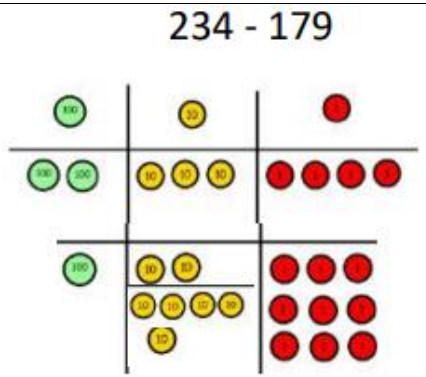
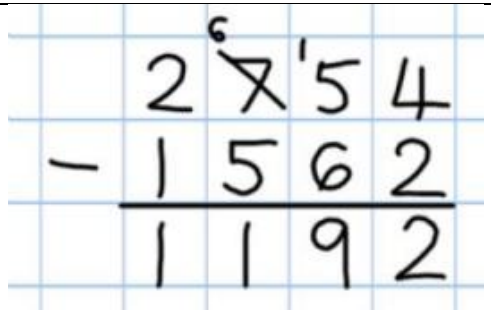
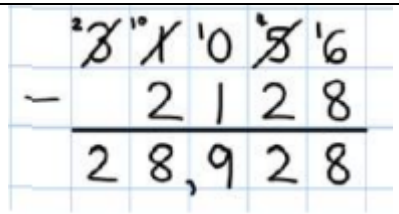
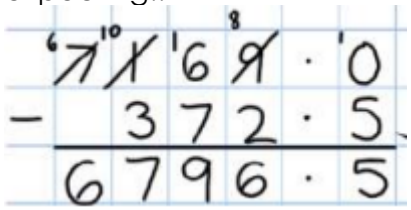
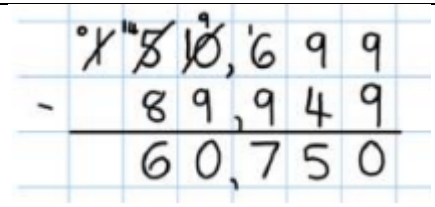
- Children should understand that subtraction is the removal of one quantity from another (not necessarily a smaller number from a greater one) or finding the **difference** between two quantities.
- Children should understand that, unlike addition, subtraction is **not** commutative.
- Ensure that children understand the = sign means ***'is the same as'*** not '*makes'* and that children see calculations where the = sign is in a different position. For example, **9 – 5 = 4** and **4 = 9 – 5**.

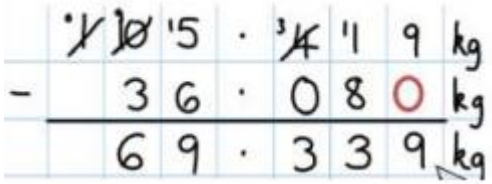
Year 3	Year 4	Year 5	Year 6
Age-related expectations: • Add and subtract numbers mentally, including: ***A three-digit number and ones*** ***A three-digit number and tens*** ***A three-digit number and hundreds*** • Add and subtract numbers with up to 3 digits, using formal written methods (columnar). • Estimate answers to calculations; use inverse to check. • Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.	**Age-related expectations:** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate. • Estimate and use inverse operations to check answers to a calculation. • Solve two-step addition and subtraction problems in context, deciding which operations and methods to use and why.	**Age-related expectations:** Add and subtract whole numbers with more than 4 digits using formal written methods of columnar addition and subtraction. • Add and subtract mentally with increasingly larger numbers. • Use rounding to check answers and determine, in the context of a problem, levels of accuracy. • Solve multi-step addition and subtraction problems in context, deciding which operations and methods to use and why.	**Age-related expectations:** • Use knowledge of the order of operations to carry out calculations involving all four operations. • Solve multi-step addition and subtraction problems in context, deciding which operations and methods to use and why. • Solve complex problems involving addition and subtraction. • Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
Vocabulary: Subtract, subtraction, minus, leave, How many are left? One less, two less, ten less, one hundred less, How many fewer is…than…? How much less is…than…? What is the difference between…and…? Half, halve, =, equals, …is the same as… How many have gone? Tens boundary, hundreds boundary	**Vocabulary:** Subtract, subtraction, minus, decrease, leave, How many are left? How many fewer is…than…? How much less is…than…? What is the difference between…and…? Half, halve, =, equals, …is the same as… Tens boundary, hundreds boundary, Inverse	**Vocabulary:** Subtract, subtraction, minus, decrease, leave How many are left? How many fewer is…than…? How much less is…than…? What is the difference between…and…? Half, halve, =, equals, …is the same as… Tens boundaries, hundreds boundary, ones boundary, tenth boundary, Inverse	**Vocabulary:** Subtract, subtraction, minus, decrease, leave How many are left? How many fewer is…than…? How much less is…than…? What is the difference between…and…? Half, halve, =, equals, …is the same as… Tens boundaries, hundreds boundary, ones boundary, tenth boundary, Inverse

Children should begin the method of expanded **decomposition** with TO – TO calculations. This process could be demonstrated using arrow cards to show the partitioning and Base 10 to show the decomposition of a number. When solving the calculation 89 – 57, children need to understand that the number being subtracted (57) does not appear as an amount on its own; rather as part of the larger amount (89). Therefore, when using Base 10, children will only need to count out the 89. 89 -57 _____ __32__ Use examples that do not involve exchanging and decomposition. Emphasise that the bottom number is being subtracted from the top number rather than the 'smaller' number being subtracted from the 'greater' number. Progress into examples that do involve decomposition. For example, 84 – 56. Children should know that ones line up under ones, tens under tens and so on. T O 7 14 ~~8~~ ~~4~~ 5 6 2 8	**NB**. Children should be encouraged to count on mentally (to find the difference) when two numbers are very close together or near to multiples of 10, 100 etc either mentally or using an empty number line with hops in efficient steps. Children should: - be able to subtract numbers with different amounts of digits. - Using formal written method, children should also begin to find the difference between two 3-digit sums of money. - Know that decimal points should all line up under each other. - When children are ready, this should lead to the formal method of **decomposition.** For example, £8.95 - £4.38 8 1 £8.~~9~~~~5~~ -£4.38 £4.57 - Children should apply their knowledge and skills of subtraction to solve word problems of increasing complexity.	Children should: - Be able to subtract numbers with different amounts of digits. - Begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places (up to 2d.p) - Know that the decimal points should line up under each other. 614 1 2 13 1 ~~17~~~~5~~4 ~~8~~.42 - 286 - 1.76 1468 1.66 - Children should apply their knowledge and skills of subtraction to solve word problems of increasing complexity.	Children should: - Be able to subtract numbers with different amounts of digits. - Be able to subtract two or decimal fractions with up to three digits and three decimal places, including units of measurement. - Know that the decimal points should line up under each other. 5 13 1 1~~6~~~~4~~67 - 2684 13783 - Children should apply their knowledge and skills of subtraction to solve word problems of increasing complexity.

Objective and strategy	Concrete	Pictorial	Abstract
Y3 Column subtraction without decomposition (friendly numbers) Column subtraction with decomposition.	Begin by using Base 10 or Numicon to model before moving on to place value counters.   Make the greatest number with the place value counters:  Start with the ones. Can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones:  Now I can subtract my ones:	Draw representations to show understanding.  Draw counters onto a place value grid and show what you have subtracted by crossing out the counters as well as clearly showing the exchanges made.  When confident, children can find their own way of demonstrating and recording the exchanging and decomposition.	Children can start their formal written method by partitioning the number into clear place value columns:  Moving forward, the children use a more compact method:  An intermediate step may be required to lead to clear subtraction understanding.  This will lead to an understanding of subtracting any number including decimals in future years:

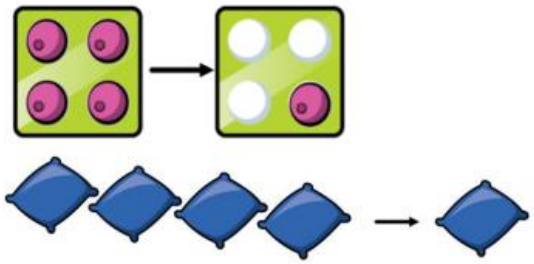
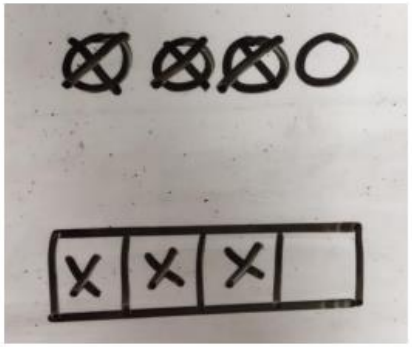
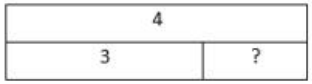
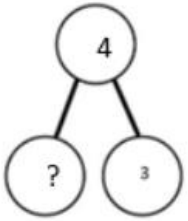
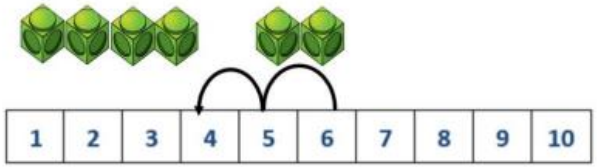
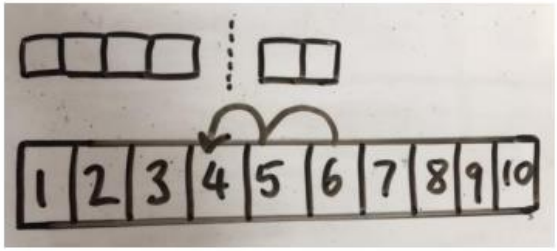
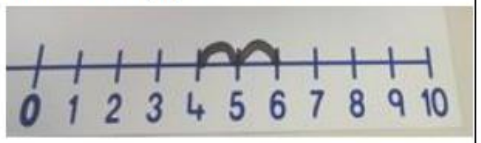
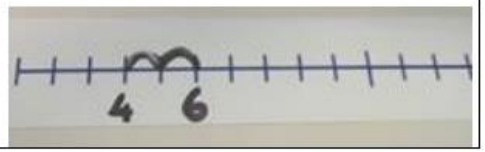
	 Now look at the tens. Can I subtract 8 tens easily? I need to exchange 1 hundred for 10 tens:  Now I can subtract 8 tens and complete my subtraction:  Show the children how the concrete method links to the written method alongside the working out. Cross out the digits when exchanging and show where we write the new amount.		
Y4 Subtract numbers with up to 4 digits.	Model the process using manipulatives and practical resources: e.g. Base 10 and Numicon, before moving on to place value counters.	Children to draw place value counters and show their exchanging and decompositions- see Year 3.	Use the vocabulary **'decomposition'** when carrying out formal written subtraction.

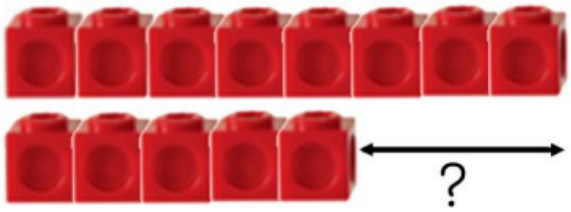
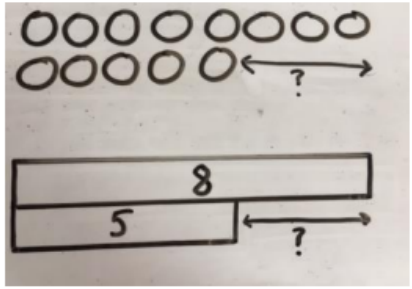
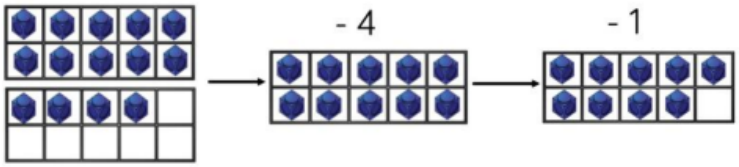
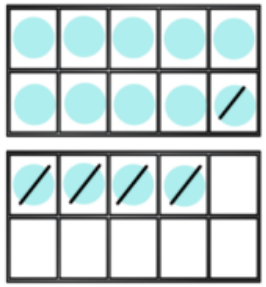
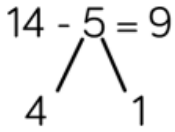
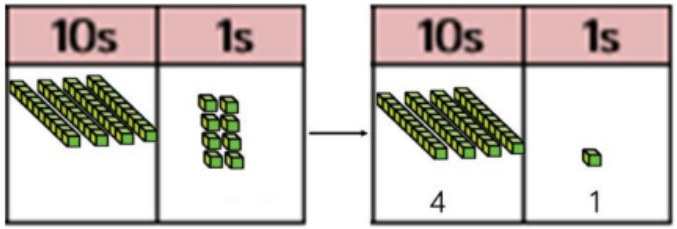
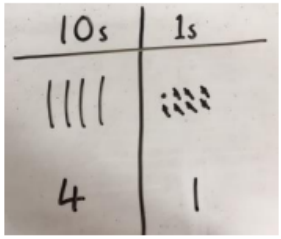
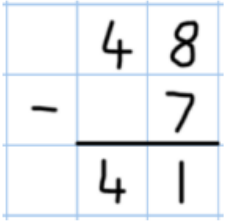
Introduce decimal subtraction through the context of money.			
Y5 Subtract numbers with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal points.*	As Year 4	Children to draw place value counters and show their exchanging and decompositions- see Year 3 and Year 4.	 In the context of decimals, use zeros for place-holders and remind children that the decimal point needs a square of its own (WRM doesn't show this but, as a school, this is what we are expecting.: 
Y6 Subtract with increasingly great and more complex	As Year 4 and 5.	Children to draw place value counters and show their exchanging and decompositions- see Year 3, Year 4 and Year 5.	

numbers and decimal values.			In the context of decimals, remind children that the decimal point needs a square of its own. 

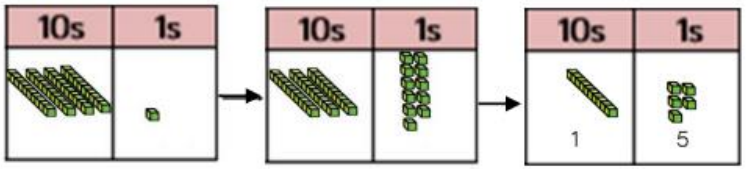
Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

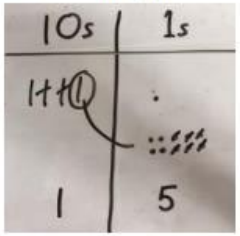
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	**4- 3 =** **□ = 4 – 3**  
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4  	Children to represent what they see pictorially e.g.  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.
Making 10 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 

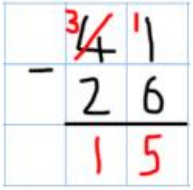
Column method using base 10 and having to exchange.
41 – 26



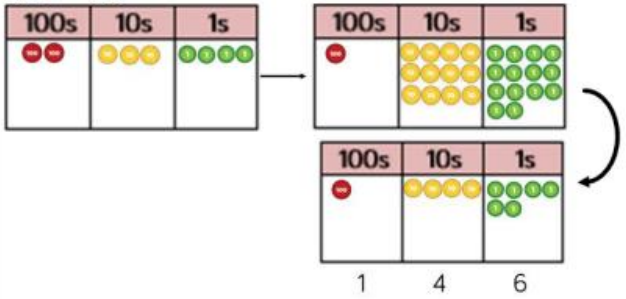
Represent the base 10 pictorially, remembering to show the exchange.



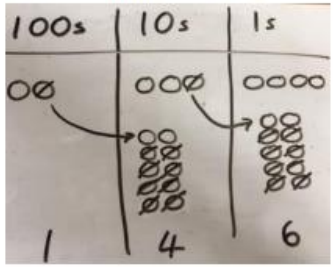
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.



Column method using place value counters.
234 – 88



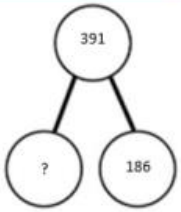
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal colum method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} {}^{2}\not{2}\,{}^{1}3\not{4} \\ -\ 88 \\ \hline 6 \end{array}$$

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

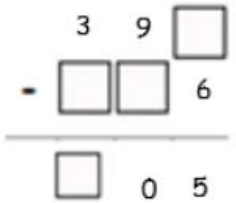
Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
———

What is 186 less than 391?

Missing digit calculations



Multiplication

- Children need to know that multiplication is commutative and use this information to rearrange calculations, knowing the 4 x 6 gives the same answer as 6 x 4.
- Children need to understand that multiplication is repeated addition.
- Ensure that children understand that the = sign means ***'is the same as'*** not '*makes'* and that children see calculations where the = sign is in a different position. For example, 3 x 5 = 15 and 15 = 3 x 5.

Year 3	Year 4	Year 5	Year 6
Age-related expectations: • Recall and use multiplication facts for the 3, 4 and 8 times tables. • Write and calculate statements for multiplication using tables they know, including for 2-digit multiplied by 1-digit numbers, using mental and progressing to formal written methods. • Solve problems, including missing number problems, involving multiplication, including integer scaling problems and correspondence problems in which n objects are connected to m objects.	**Age-related expectations:** • Recall Recall multiplication facts up to 12 x 12. • Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers. • Recognise and use factor pairs and commutativity in mental calculations. • Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. • Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.	**Age-related expectations:** • Identify multiples and factors; find all factor pairs of a number and common factors of 2 numbers. • Know & use the vocabulary of prime numbers, prime factors & composite numbers. • Establish whether a number up to 100 is prime; recall primes up to 19. • Multiply numbers up to 4 digits by a one or two-digit number using a formal method, including long multiplication for two-digit numbers. • multiply and divide numbers mentally drawing upon known facts • Multiply and divide whole numbers and those involving decimals by 10, 100 & 1000. • Recognise and use square numbers & cube numbers and notation for squared 2, cubed 3 • Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	**Age-related expectations:** As year 5 plus: **ALGEBRA:** • Use simple formulae • Generate and describe linear number sequences • Express missing number problems algebraically • Find pairs of numbers that satisfy number sentences involving two unknowns • Enumerate possibilities of combinations of two variables. **RATIO AND PROPORTION** • Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts • Solve problems involving the calculation of percentages (for example, of measures, and such as 15% of 360) and the use of percentages for comparison • Solve problems involving similar shapes where the scale factor is known or can be found

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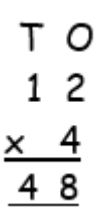
Solve problems involving + - x ÷ and a combination of these, including understanding meaning of = sign
Solve problems involving x and ÷ including scaling by simple fractions & problems involving simple rates.

</td>
<td>

Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

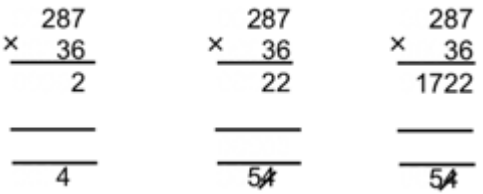
</td>
</tr>
<tr>
<td><u>Vocabulary:</u>
Multiplication, multiply, multiple, factor, groups of, times, product,
Once, twice, three times…ten times (the size of…),
Repeated addition, grouping, equal groups of, array, row, column, multiplication table, multiplication fact.</td>
<td><u>Vocabulary:</u>
Multiplication, multiply, multiple, factor, groups of, product,
Once, twice, three times…ten times (the size of…),
Repeated addition, grouping, equal groups of, doubling, array, row, column, multiplication fact, number patterns, inverse, square number, squared, cube number, cubed.</td>
<td><u>Vocabulary:</u>
Multiplication, multiply, multiplied by, multiple, factor, groups of, produce,
Once, twice, three times…ten times (the size of…),
Repeated addition, grouping, equal groups of, doubling, array, row, column, multiplication fact, number patterns, inverse, square number, squared, cube number, cubed.</td>
<td><u>Vocabulary:</u>
Multiplication, multiply, multiplied by, multiple, factor, groups of, product,
Once, twice, three times…ten times…one hundred times (the size of…),
Repeated addition, grouping, equal groups of, doubling, array, row, column, multiplication fact, number patterns, inverse, squared number, squared, cube number, cubed.</td>
</tr>
<tr>
<td>Children should be confident with mental calculations (for example, 15 x 5) by using known table facts; e.g: 10 x 5 =50, 5 x5 = 25, therefore, 50 + 25 = 75. They should also be comfortable using standard written methods of addition before they approach 2-digit x 1-digit multiplications.

Introduce 2-digit x 1-digit without carrying:

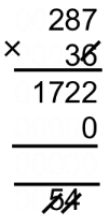


Starting with the ones
2 x 4 = 8.
8 is placed in the ones column.
1T x 4 = 4T placed in the tens column
So 12 x 4 = 48

This method can be demonstrated by using practical resources (e.g. Dienes apparatus) to ensure understanding.</td>
<td>As Year 3, except children should consolidate the formal written layout by using 3-digit numbers multiplied by 1-digit, involving carrying and crossing the hundreds boundary</td>
<td colspan="2">Children should be confident using a standard written method to multiply 3 or 4-digit numbers by 1-digit with carrying and multiplying decimal numbers by 1-digit numbers (e.g. money and measures) before progressing to long multiplication. For example, 287 x 36:



When x by a 2 digit number you will need to make two lines of calculations. Ignore the digit in the T column (3).
Start with multiplying each digit of the top number by the one of the bottom number.
6x7=42. The 2 is in the first line, in the O column. The 4 is in the T column under the bottom line.
Repeat with 8 x 6 then 2 x 6 adding the carrying digits as you go along.
So 287 x 6 = 1722



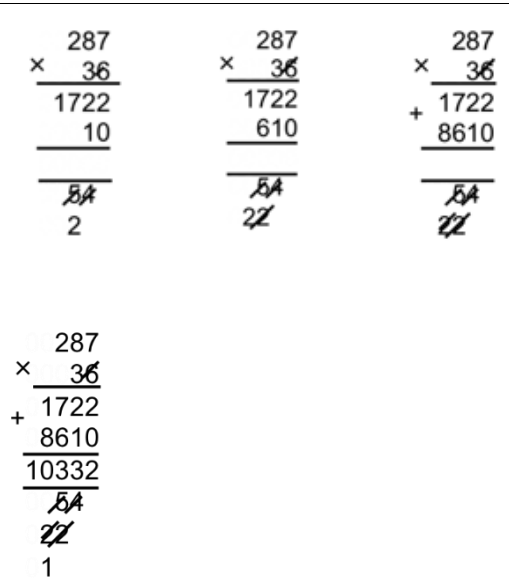
The next IMPORTANT step is that a place holder is placed on the second answer line, in the O column. This is to show that the next set of multiplications are multiplied by 10's. By putting the zero in first, all answers are effectively moved one place to the left and so multiplied by ten.</td>
</tr>
</table>

Leading to:

H	T	O
	2	4
×		6
1	4	4
	2	

Starting with the ones,
4 x 6 = 24.
4 is placed in the ones column and the 2T are carried under the line.
2 T x 6 = 12 T and we need to + on the 2T we have already carried = 14 T
14 T = 1 H and 4 T
So 24 x 6 = 144

This method can be demonstrated using apparatus ensuring the children are exchanging appropriately. Encourage children to strike through the carrying digit.

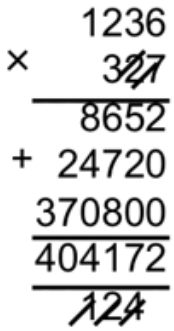


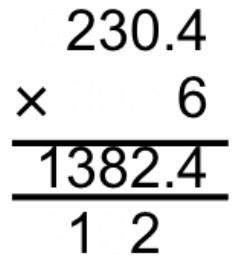
This, now, allows the calculation to be completed by multiplying by 3.
7 x 3 = 21 , 1 is placed in the next available space (T) and the 2 in the H column.
8 x 3 = 24 + 2 (carried from the previous calculation) = 26. The 6 is placed in the H column and the 2 carried to under the Th column.
2 x 3 = 6 + 2, (carried from the previous calculation) = 8.

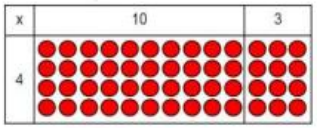
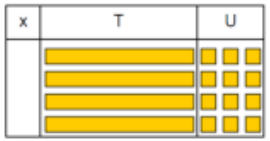
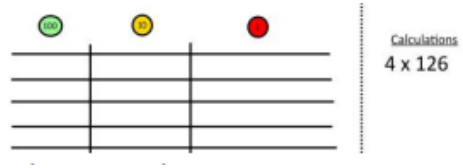
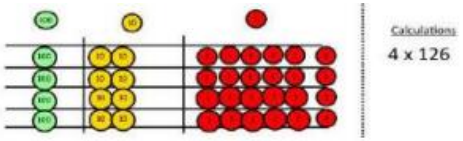
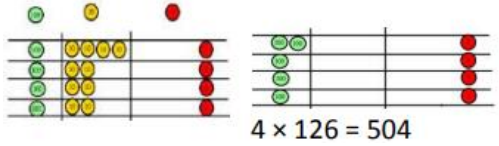
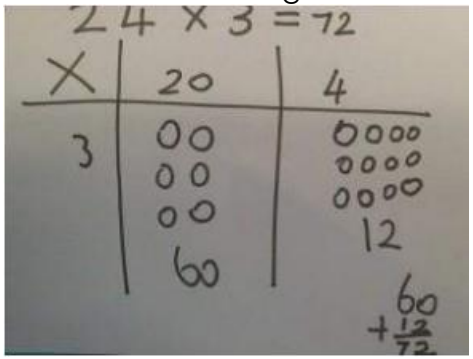
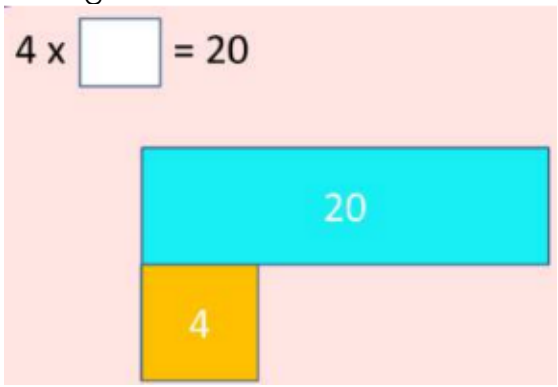
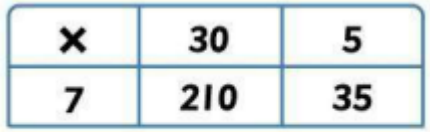
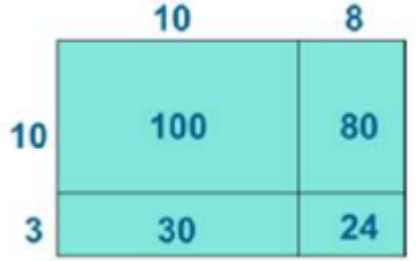
So 30 x 287 = 8610.

The final step is to add the two partial products together using prior knolwedge of stabdard method for addition, so 287 x 36 = 10, 332

Year 6 build on calculation methods learned in Year 5 by multiplying 4-digit numbers by 3-digit numbers and multiplying decimal numbers by whole numbers.

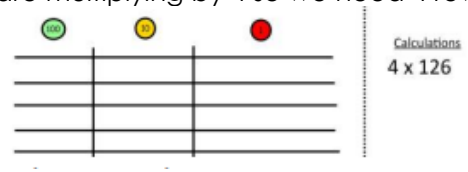


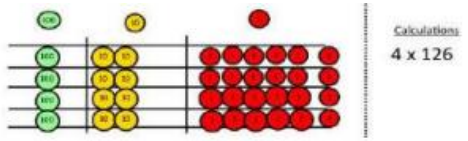


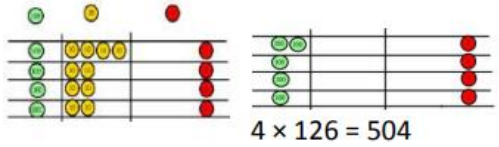
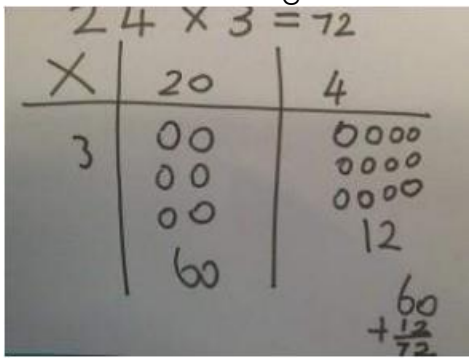
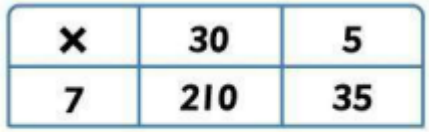
Objective and strategy	Concrete	Pictorial	Abstract
Y3 Grid method and arrays	Show the links with arrays to first introduce the grid method:  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method:  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows:  Fill each row with 126:  Add up each column, starting with the ones; making any exchanges where needed:  	Children can represent their work with place value counters in a way they understand. They can draw the counters using colours to show different amounts or just the circles in the different columns to show their thinking:  Bar models can be used to explore missing numbers:  	Start with multiplying by one-digit numbers and showing the clear addition alongside the grid:  210 + 35 = 245 Moving forward, multiply by a 2-digit number showing the different rows within the grid method: 

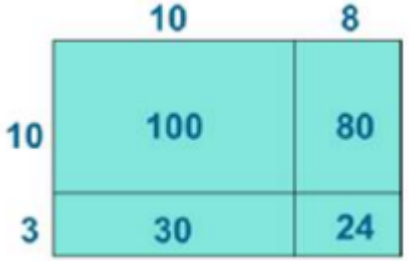
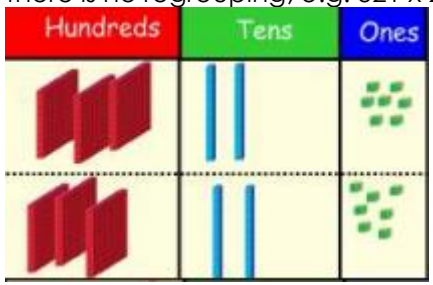
<table>
<tr>
<td>Y4

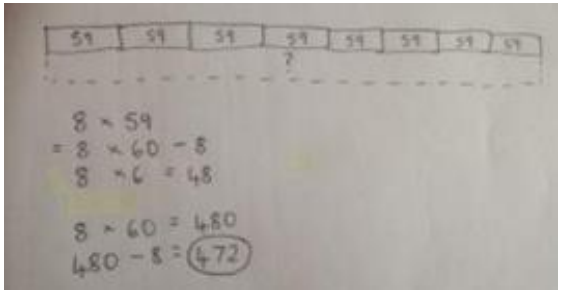
Grid method recap from Year 3 for 2-digits x 1-digit.

Move to multiplying 3-digits by 1-digit (Year 4 expectations)</td>
<td>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows:
 

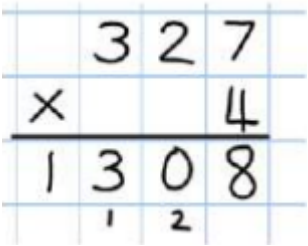
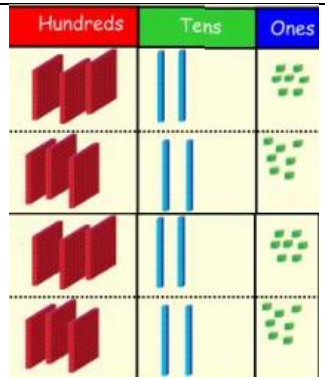
Fill each row with 126:
 

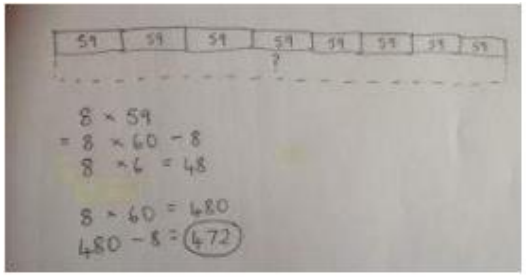
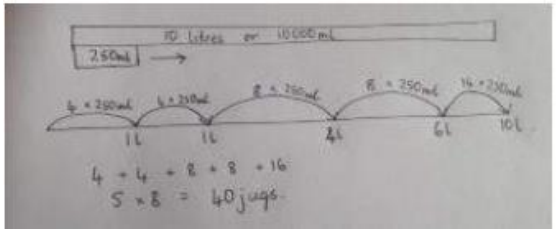
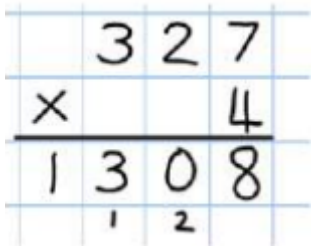
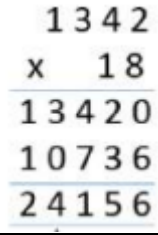
Add up each column, starting with the ones; making any exchanges where needed:
 
</td>
<td>Children can represent their work with place value counters in a way they understand. They can draw the counters using colours to show different amounts or just the circles in the different columns to show their thinking:
 
</td>
<td>Start with multiplying by one-digit numbers and showing the clear addition alongside the grid:
 

210 + 35 = 245
Moving forward, multiply by a 2-digit number showing the different rows within the grid method:
 
</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at this stage of multiplication. This, initially, is done where there is no regrouping; e.g. 321 x 2 = 642:
 

It is important at this stage that the children always multiply the ones first. The corresponding long multiplication is modelled alongside.</td>
<td>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.
 

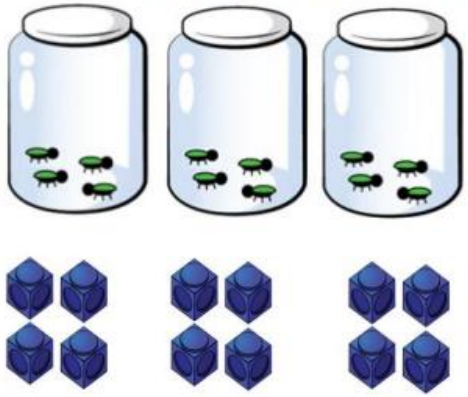
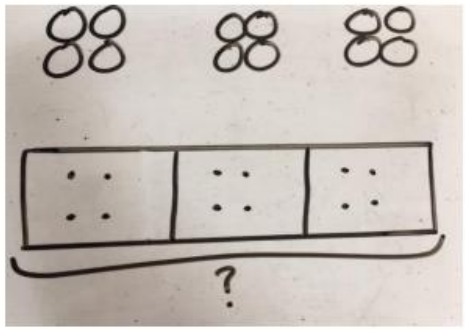
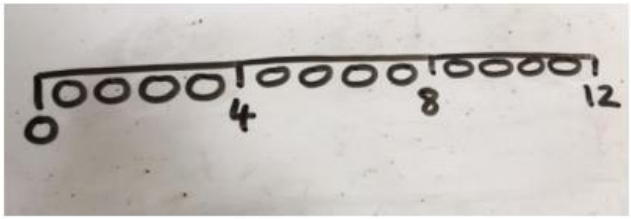
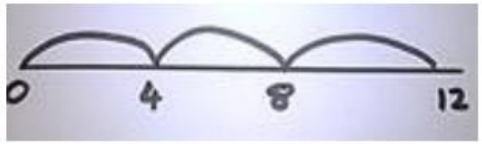
The grid method may be used to show how this relates to a formal written method</td>
<td></td>
</tr>
</table>

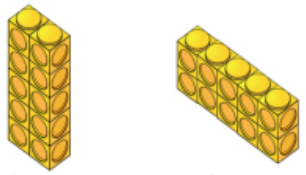
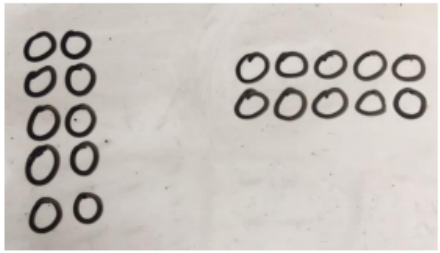
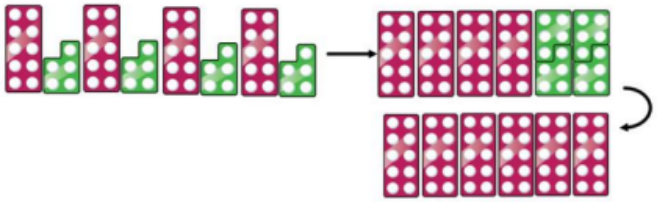
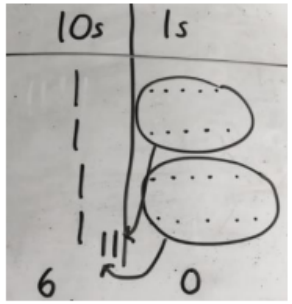
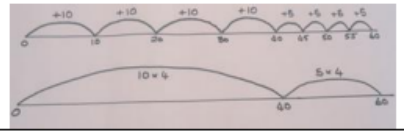
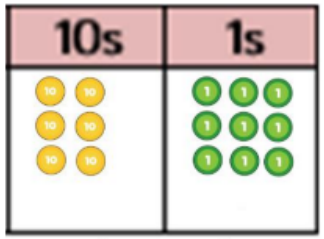
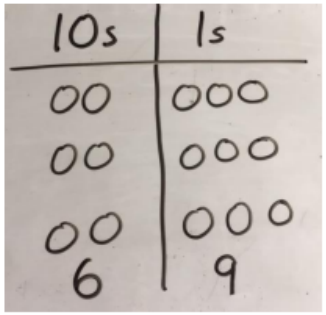
		<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>	327 x 4 28 80 1200 1308 
Y5 / Y6 Column multiplication for 3 and 4- digits multiplied by 1-digit	 It is important at this stage that children always multiply the ones first. They can continue to be supported by place	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>	327 x 4 28 80 1200 1308

Column multiplication.	value counters at this stage of multiplication. Manipulatives can still be used with the corresponding long multiplication modelled alongside.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods:   	 Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer:  This moves to the more compact method of multiplication: 

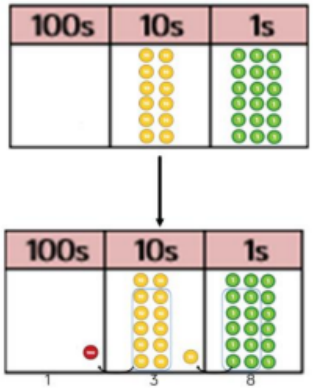
Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

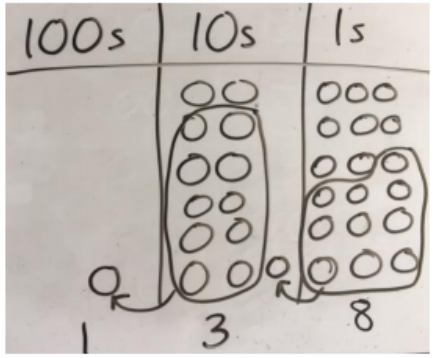
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 

Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3×23  6 9	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 20 3 $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \end{array}$

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

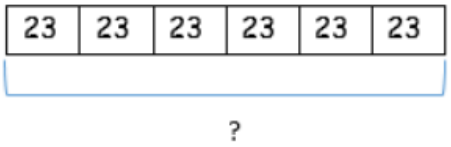
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

□ = 6 × 23

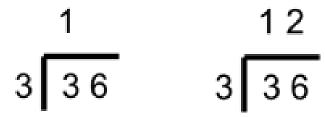
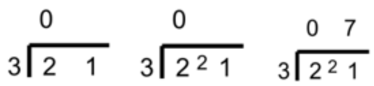
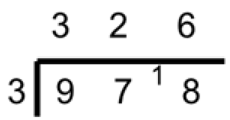
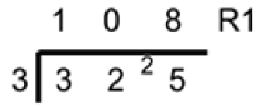
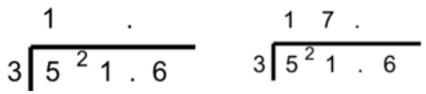
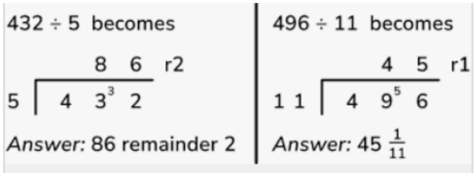
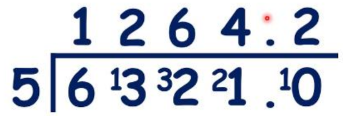
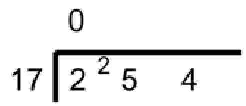
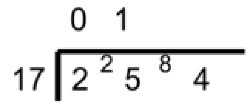
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?

Division

- Children need to understand that division can be **sharing, grouping** or **repeated subtraction**.
- Children should understand that division is **not commutative**.
- Ensure that children understand the = sign means ***'is the same as'*** as opposed to '*makes*' and that children see calculations where the = sign is in a different position. For example, **12 ÷ 3 = 4** and **4 = 12 ÷ 3**.

Year 3	Year 4	Year 5	Year 6
Age-related expectations: • Recall and use ÷ facts for the 3, 4 and 8 times tables. • Write and calculate statements for ÷ using tables they know, including for 2-digit ÷ 1-digit using mental and written methods. • Solve problems, including missing number problems, involving division, including integer scaling problems and problems in which objects are connected to other objects.	**Age-related expectations:** • Recall division facts up to the 12 times tables. • Use place value and known facts to divide mentally, including dividing by 1. • Recognise and use factor pairs and commutativity in mentally calculations. • Solve problems involving dividing, including integer scaling problems and more challenging problems in which objects are connected to other objects.	**Age-related expectations:** • Identify multiples and factors- find all factor pairs of a number and common factors. • Know and use the vocabulary of prime numbers, prime factors and composite numbers. • Recall prime numbers up to 19 and establish whether a number up to 100 is prime. • Divide numbers mentally using known facts. • Divide numbers up to 4-digits by 1-digit numbers using formal written methods and using remainders. • Divide numbers by 10, 100 and 1000. • Solve division problems using knowledge of factors, multiples, squares and cubes. • Solve problems using all 4 operations. • Solve problems involving division, including scaling by simple fractions.	**Age-related expectations:** As year 5 plus: **ALGEBRA** • Use simple formulae. • Generate and describe linear number sequences. • Express missing number problems algebraically. • Find pairs of numbers that satisfy number sentences involving 2 unknowns. • Enumerate possibilities of combinations of two variables. **RATIO AND PROPORTIONS** • Solve problems involving the relative sizes of two quantities where missing values can be found using x and ÷ facts. • Solve problems involving the calculation of percentages (for example, 15% of 360) and the use of percentage for comparison. • Solve problems involving similar shapes where the scale factor is known or can be found. • Solve problems involving unequal sharing using knowledge of fractions, factors and multiples.

Vocabulary: Half, halve, share, share equally, One each, two each, three each… Group in pairs, threes…tens, Equal groups of… ÷, divide, divided by, division, dividend, divisor, Left, left over, remainder	**Vocabulary:** Half, halve, share, share equally, One each, two each, three each… Group in pairs, threes…tens, Equal groups of… ÷, divide, divided by, division, dividend, divisor, quotient Left, left over, remainder Factor, inverse	**Vocabulary:** Half, halve, share, share equally, One each, two each, three each… Group in pairs, threes…tens, Equal groups of… ÷, divide, divided by, division, dividend, divisor, quotient Left, left over, remainder Factor, inverse	**Vocabulary:** Half, halve, share, share equally, One each, two each, three each… Group in pairs, threes…tens, Equal groups of… ÷, divide, divided by, division, dividend, divisor, quotient Left, left over, remainder Factor, inverse
Children should be confident using times tables facts to assist with division before short division (bus stop) method is introduced: 36 ÷ 3 = 12  Start with the tens digit: 3 ÷ 3 = 1. The answer is being placed above the dividend being looked at. Next, the ones: 6 ÷ 3 = 2 which is put above the dividend. 21 ÷ 3 = 7  If the dividend is too small for the divisor to divide into it a zero is placed over the dividend. The two hasn't been divided yet; it must be carried over to the next digit. In this case a 1. It becomes 21 again. 21 ÷ 3 = 7	Children consolidate previous learning on short division, extending to 3-digit ÷ 1-digit numbers with and without remainders. Money and measures can be included so that children are calculating with simple decimal numbers divided by a whole number. 978 ÷ 3 = 326  325 ÷ 3 = 108 r1  51.6 ÷ 3 = 17.2 	In year 5, children consolidate previous learning on short division, extending to 4-digit ÷ 1-digit with and without remainders. Money and measures can be included so that children are calculating with simple decimal numbers divided by a whole number. Children begin to convert remainders into fractions. The remainder becomes the numerator and the divisor becomes the denominator:  	Introduce the concept of converting the remainder into a fraction and a decimal by using place holders:  Introduce long division (3 and 4-digit numbers divided by 2-digit numbers). 254 ÷ 17 = 14 r16  When the divisor is a two-digit number, the first dividend will always have a zero over it. **TIP:** write out the first 5 or so multiples of the divisor- this can be extended if needed.  17, 34, 51, 68, 75 25 ÷ 17 = 1 with 8 remaining.

28 ÷ 3 = 9r1

$$\begin{array}{r} 0\ 9\ \text{R1} \\ 3\overline{)\,2^{2}8} \end{array}$$

The same process as above is carried out.

$$\begin{array}{r} 1\ 7\ .\ 2 \\ 3\overline{)\,5^{2}1\ .\ 6} \end{array}$$

When the dividend is a decimal, line up the decimal point in the answer.

5 ÷ 3 = 1 with 2 left over, the two left over is carried over to the next digit, becoming 21.
21 ÷ 3 = 7
6 ÷ 3 = 2
So 51.6 ÷ 3 = 17.2

$$\begin{array}{r} 0\ 1\ \ 4 \\ 17\overline{)\,2^{2}5^{8}4} \end{array}$$

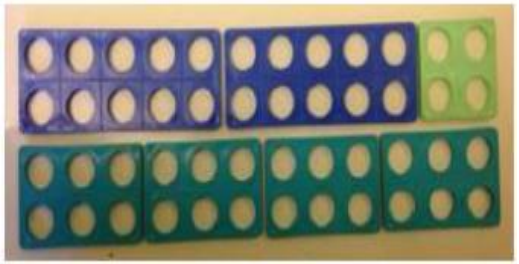
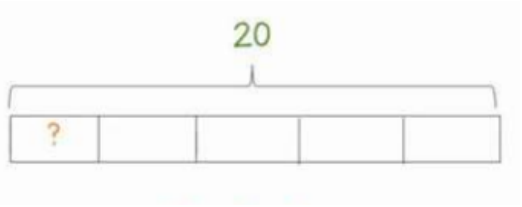
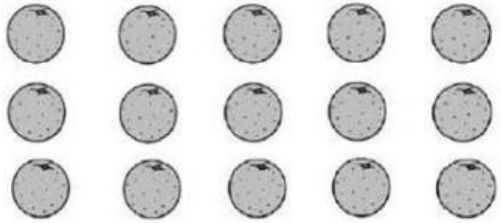
84 ÷ 17 = 5 with 16 remaining.

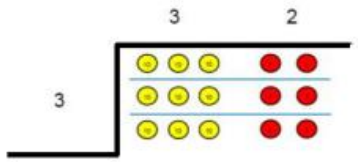
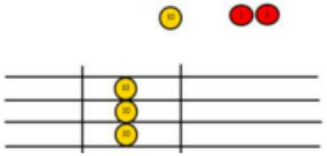
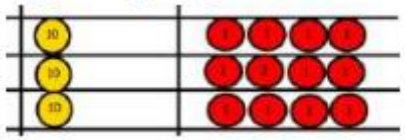
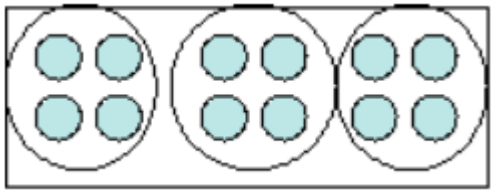
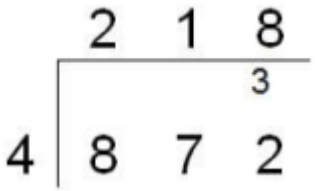
$$\begin{array}{r} 0\ 1\ \ 4\ \ \text{R16} \\ 17\overline{)\,2^{2}5^{8}4} \end{array}$$

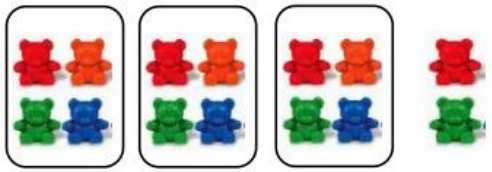
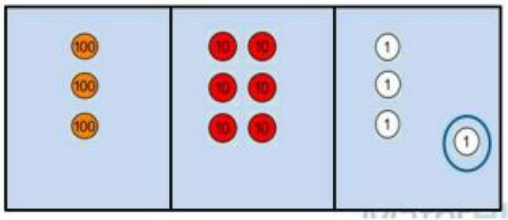
Begin to introduce the formal written method of long division. For example:

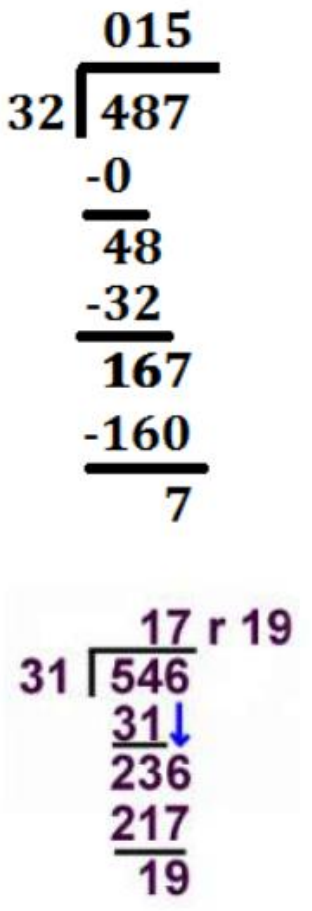
543 ÷ 24 = 543

$$\begin{array}{r} 5\,4\,3 \\ 24\overline{)\,1\,3\,0\,3\,2} \\ -\,1\,2\,0\downarrow\ \ \ \ \\ \quad 1\,0\,3\ \ \ \\ \quad -\,9\,6\ \ \ \\ \quad\ \ 7\,2\ \ \ \\ \quad -\,7\,2\ \ \ \end{array}$$

Objective and strategy	Concrete	Pictorial	Abstract
Y3/4 Division as grouping	Use cubes, counters, objects or place value counters to aid understanding of division.  24 divided into group of 6 = 4.	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 3 = 5 15 ÷ 5 = 3 15 = 3 x 5 15 = 5 x 3 5 = 15 ÷ 3 3 = 15 ÷ 5
Short division	Use to place value counters to divide using the short division method alongside. 96 ÷ 3	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.	Begin with divisions that divide equally with nor remainder.

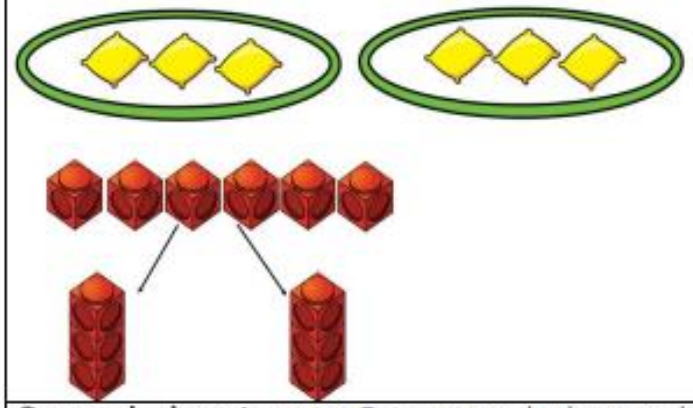
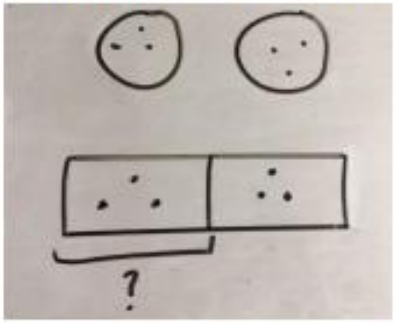
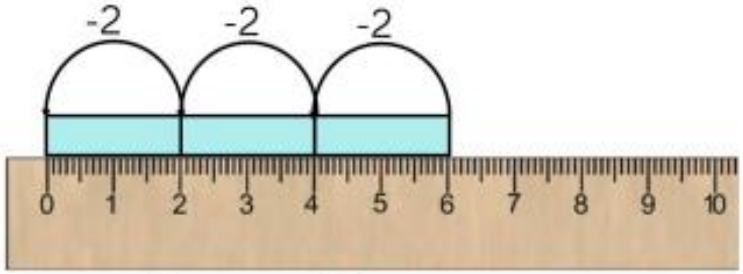
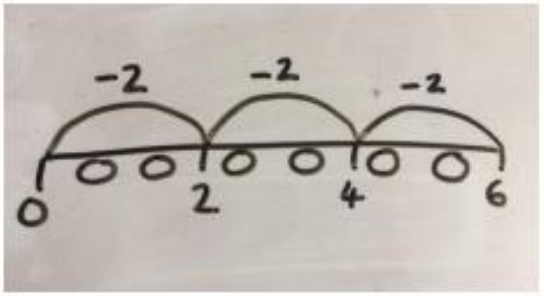
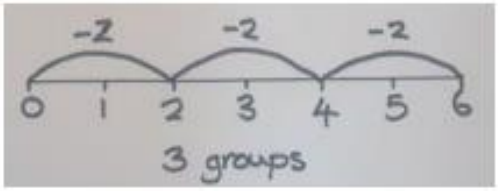
	 42 ÷ 3   Start with the biggest place value. We are sharing 40 into three groups. We can put one 10 in each group and we have one 10 left over.  We exchange this ten for 10 ones and then share them equally among the groups. We look at how many are in each group. 	 Encourage them to move towards counting in multiples and divide more efficiently.	
Y5/6 Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over.	Jump forward in equal groups on a number line then see how many more you need to jump to find a remainder. 	Complete written divisions and show the remainder using 'r'. 

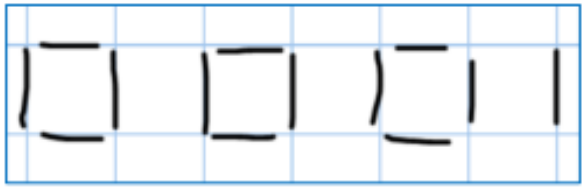
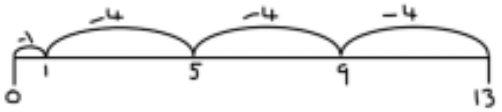
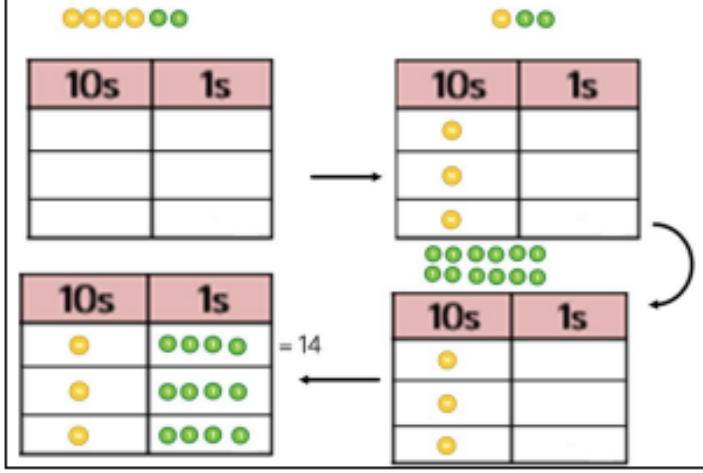
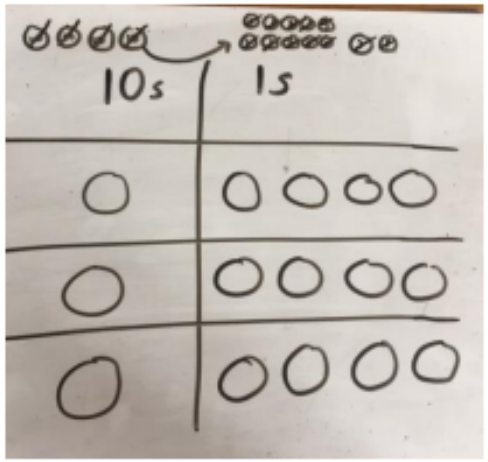
		Draw dots and group them to divide an amount and clearly show a remainder. 	
Short division with remainders	364 ÷ 3 = 1 2 1 rem 1 3 | 3 6 4 		Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or a decimal according to the context. 8 6 r 2 5 | 4 3 2 1 8 6 1/5 5 | 9 4 3 3 1 1 4 . 6 3 5 | 5 1 1 . 0
Y6			Children will use long division to divide numbers with up to 4-digits by 2-digit numbers.

Long division			

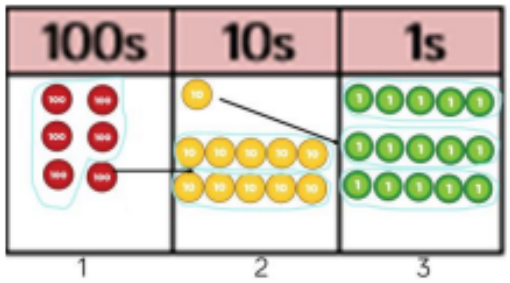
Calculation policy: Division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

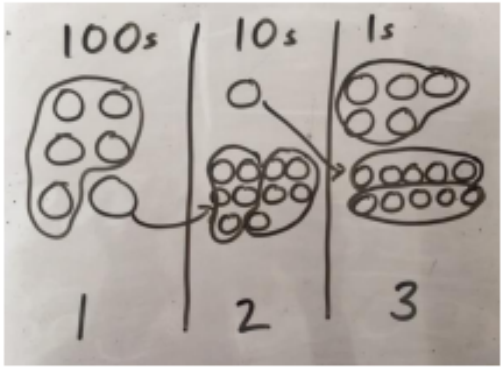
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

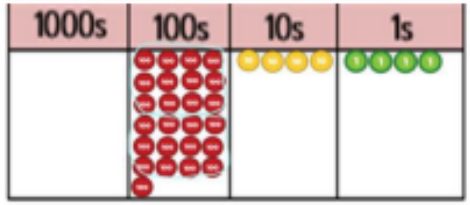


Children to the calculation using the short division scaffold.

$$5\overline{)615}\quad 123$$

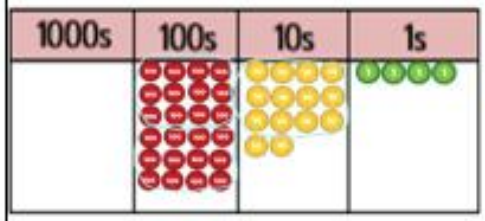
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



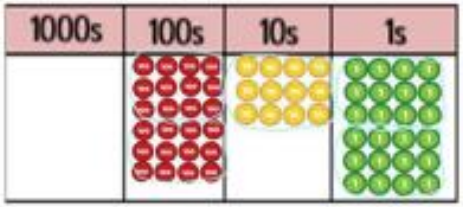
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

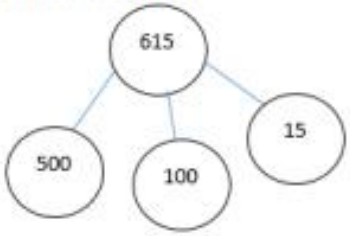


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$5\overline{)615}$

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?
What is the answer?